

FUZZY SETS APPLICATION IN THE PROBLEMS OF STRUCTURAL MECHANICS AND OPTIMAL DESIGN

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1. Introduction

In the practical design and study of load-bearing structures, there exist situations when information is uncertain and part of the source data is specified randomly, roughly or fuzzy. Therefore, along with deterministic statements, it is of interest to consider more general problems in the design theory, including the optimal one, which would take into account information that has a certain degree of uncertainty about the source data, environment behavior, goals, constraints, etc.

For their formulation and solution, a corresponding mathematical apparatus is required, which would a priori include the possibility of the appearance of uncertainty [2, 3]. The theory of probability and statistical methods are widely used [1, 6, 7] to take into account random factors in structural mechanics. Here, a priori, statistical descriptions of random variables are assumed to be known and reliable. They are the probability distribution function, probability density function and their characteristics. Their estimates are usually obtained as a result of processing of a large statistical sample. In the case when there is a small statistical sample, when the description of parameters, goals, restrictions are set subjectively or verbally, it is possible to design by using the apparatus of the fuzzy sets theory (FST) L. Zadeh [10, 13, 16]. It forms the basis of the theory of possibilities [8], “soft” computations [15], and fuzzy modeling [11]. In this paper, the example of the analysis problems of a bent beam and the optimal design of a minimum volume truss shows the methodology for applying FST. Numerical experiments were performed for various degrees of the initial information fuzziness. An analysis of the “tolerance” of the final results to the “blurred” data is carried out by comparison with the design of the structure with clear initial information. To implement the deterministic model of truss optimization, an iterative computational procedure was developed based on the functional equation of the Bellman method [5]. Fuzzy modeling previously was considered by the authors to estimate the maximum value of the axial compression force of a cylindrical shell with fuzzy data [4].

The approach of FST to the problems of analysis and optimization makes it possible to use it at the stage of conceptual design of technical systems.

2. Statement of the problem

Deterministic approach

The problems of analysis and optimal design in structural mechanics are considered as

$$f : X \rightarrow Y, \quad (1)$$

where f is some reflection of the elements of the set X to the set Y . This reflection can be: real-valued functions, equations, algorithms, programs of computational complexes. Set X is formed from deterministic as well as uncertain elements of a random, fuzzy and rough nature. Elements of set Y accordingly will be the same nature. Let's consider the problem of optimal design of the elastic hinge-rod system (HRS) - beam truss of minimum volume with length L in condition of strength and normal vibration restrictions. System has m hinged joints and n rods. Their lengths and cross sections are $l_i, A_i; i = \overline{1, n}$. The structural system is loaded with $P_j; j = \overline{1, m}$ in the according joints (Fig. 1).

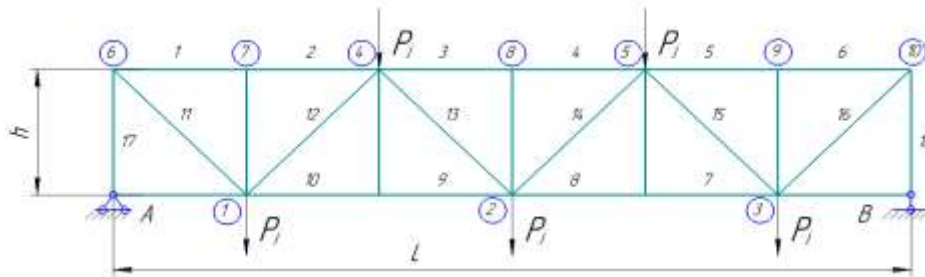


Fig. 1. Design scheme of HRS

It is necessary to find a distribution of material $l_i A_i; i = \overline{1, n}$ in the elements of the truss in such conditions as:

- strength and stability restriction

$$\sigma_i = \frac{N_i}{A_i} \leq R_i^*; \quad R_i^* = \begin{cases} R_0, N_i > 0 \\ \varphi R_0, N_i < 0 \end{cases}; \quad (2)$$

- normal vibration frequency do not exceed a predetermined value ω_0

$$\omega(A_i) \geq \omega_0 \quad (3)$$

and at the same time the minimum value of the volume of the material of the whole structure was ensured

$$V = \sum_{i=1}^n l_i A_i \rightarrow \min. \quad (4)$$

In restriction (2) indicated the N_i what is a longitudinal force in i element, R_0 - characteristic tensile strength of the material, φ - buckling coefficient.

The mathematical model of the formulated deterministic optimization problem is

$$(V^{opt}, A^{opt}) = \arg \left\{ \min_{A_i \in \Omega_i} \sum_{i=1}^n l_i A_i \mid \omega(A_i) \leq \omega_0 \right\}, \quad (5)$$

where $A^{opt} = (A_1^{opt}, A_2^{opt}, \dots, A_n^{opt})$ is an optimal solution vector.

The problem is to find the desired reflection f , that is, to develop a computational procedure for finding the result $Y = (V^{opt})$ on the base of deterministic data $X = (l_i, i = 1, 2, \dots, n; P_j; j = 1, 2, \dots, m; E, R_0, \varphi, \omega_0)$ in order to introduce it into fuzzy modeling technology.

Feasible region $\{A_i\}$; $i = \overline{1, n}$, is determined by the conditions of strength and stability (2) as:

$$\Omega_i = \left\{ A_i \mid A_i \geq A_i^-; A_i^- = |N_i| / R_i^* \right\}. \quad (6)$$

The value of the normal vibration frequency of the truss will be determined approximately in accordance to the work [9]. In this case, the consideration of the rod system (Fig. 1) is replaced by the supported beam with length L and with some uniformly distributed load q . The maximum deflection of the beam in this case is as:

$$y^{\max} = \frac{5}{384} \frac{ql^4}{EI}, \quad (7)$$

where E is Young's modulus; I - moment of inertia of the beam cross section; Maximum displacement Δ^{\max} of the hinged joint 2 in the middle of the truss can be calculated with Mohr formula

$$\Delta^{\max} = \sum_{i=1}^n \frac{N_i \overline{N}_i l_i}{EA_i}, \quad (8)$$

where \overline{N}_i are the longitudinal forces in the elements of the truss from the action of a unit load applied to the joint t . Out of equality

$$y^{\max} = \Delta^{\max} \quad (9)$$

if we take in to account (7) - (8), the moment of inertia can be described as

$$I = \frac{5}{384} \frac{ql^4}{E\Delta^{\max}} \quad (10)$$

The fundamental normal vibration frequency of free transverse oscillations of a hinge-supported beam is determined as follows

$$\omega = \frac{\pi^2}{l^2} \sqrt{\frac{EI}{m}}, \quad (11)$$

where m is a mass of the beam.

If we substitute equation (10) of the moment of inertia I to the equation (11) then

$$\omega = \lambda \sqrt{\frac{g}{\Delta}}; \quad \lambda = \pi^2 \sqrt{\frac{5}{384}}, \quad (12)$$

where g is the acceleration of gravity. In the case when the fundamental normal vibration frequency is given by ω_0 then from formula (12) we obtain

$$\Delta_0 = \left(\frac{\lambda}{\omega_0} \right)^2 g. \quad (13)$$

Problem (5) with account of definitions (6), (8), (9), (13) can be formulated in the following form

$$(V^{opt}, A^{opt}) = \arg \left\{ \min_{A_i \in \Omega_i} \sum_{i=1}^n l_i A_i \mid \sum_{i=1}^n \frac{D_i}{A_i} \leq \Delta_0 \right\}, \quad (14)$$

where $D_i = N_i \overline{N}_i l_i / E$.

If we enter the following designations: $x_1 = L$, $x_2 = E$, $x_3 = \omega_0$, $x_4 = R_0$, $x_5 = \varphi$, $x_j = P_j$; $j = \overline{6, m}$; $x_j = l_j$; $j = \overline{m+1, m+n}$; $y_1 = V^{opt}$; $y_i = A_i^{opt}$; $i = \overline{2, n+1}$, then values x_j ; $j = \overline{1, m+n}$ form set of elements X , and values y_i form set Y . So the reflection f from (1) is an implementation algorithm of deterministic model (14). This algorithm will be an analytical-numerical procedure of step by step method and dynamic programming [5].

3. Dynamic programming (DP)

Model (14) in mathematical programming belongs to the class of so-called "resource distribution" problems, to which the dynamic programming method is adapted. In accordance with this method, we introduce into consideration the Bellman function.

$$f_i(d_i) = \min_{A_i, A_{i+1}, \dots, A_n} \sum_{j=i}^n l_j A_j. \quad (15)$$

where $f_i(d_i)$ - minimum volume value of HRS assuming the search process $\{A_i^{opt}\}$; $j = i, i+1, \dots, n$ begins from element $j=i$ and ends on element $j=n$. The argument of this function is the value of the "resource" d_i , which is related to the design variable A_i with such ratio as

$$d_i = \sum_{j=i}^n D_j / A_j; \quad i = 1, 2, \dots, n; \quad d_i \geq 0; \quad d_1 = [y]. \quad (16)$$

where $[y] = \Delta_0$ is determined with the ratio (13) and has a name as prior value of "resource".

From the definition (16) follows

$$d_i = \frac{D_i}{A_i} + \sum_{j=i+1}^n D_j / A_j \rightarrow d_i = \frac{D_i}{A_i} + d_{i+1}, \quad d_{i+1} = d_i - \frac{D_i}{A_i}. \quad (17)$$

Taking into account the definition (15), we construct the equation:

$$f_i(d_i) = \min_{A_i} \left[l_i A_i + \min_{A_{i+1}, A_{i+2}, \dots, A_n} \sum_{j=i+1}^n l_j A_j \right],$$

from which it follows

$$f_i(d_i) = \min_{A_i} [l_i A_i + f_{i+1}(d_{i+1})]; \quad i = n-1, n-2, \dots, 1; \quad d_1 = [y]; \quad f_{n+1}(d_{n+1}) \equiv 0 \quad (18)$$

Equation (18) is derived from the Bellman principle. It is the main functional equation of the dynamic programming method. In the terminology of this method the variable d_i is a "state" variable, and A_i is a variable of "control" (design).

From the definitions (17) and (18) for $i = n$ we have

$$d_n = D_n / A_n; A_n = D_n / d_n; f_n(d_n) = \min_{A_n} l_n A_n = l_n D_n / d_n. \quad (19)$$

Let's consider the case when $i = n - 1$. Equation (18) for this case has the form

$$f_{n-1}(d_{n-1}) = \min_{A_{n-1}} [l_{n-1} A_{n-1} + f_n(d_n)]; d_n = d_{n-1} - D_{n-1} / A_{n-1}. \quad (20)$$

With account of the definition (19), we rewrite equation (20) in the form

$$f_{n-1}(d_{n-1}) = \min_{A_{n-1}} \left[l_{n-1} A_{n-1} + \frac{l_n D_n}{d_{n-1} - D_{n-1} / A_{n-1}} \right].$$

Performing further similar procedure for $i = n - 2, n - 3, \dots, 1$ inductively with account of $f_{n-1}(d_{n-1}), f_{n-2}(d_{n-2}), \dots$, we get the following equation

$$f_i(d_i) = \min_{A_i} H(A_i), \quad (21)$$

where

$$H(A_i) = l_i A_i + T_i + \frac{D_n l_n}{d_i - S_i^* - D_n / A_i}; \quad (22)$$

$$T_i = \sum_{j=i}^{n-1} l_j A_j^*, \text{ for } i = 1, 2, \dots, n-2. S_i^* = \begin{cases} \sum_{j=i}^{n-1} D_j / A_j^*, & \text{for } i = 1, 2, \dots, n-2 \\ 0, & \text{for } i = n-1 \end{cases}$$

$\{A_j^*\}$ - an "optimal" solution on steps $i = 1, 2, \dots, n-1$.

Application of the necessary condition for the existence of an extremum of a function $H(A_i)$ from (21) - (22) gives the following equation for variable A_i on the step i

$$l_i A_i^2 - \frac{D_n l_n D_i}{(a_i^* A_i - D_i)^2} = 0, \quad (23)$$

where $a_i^* = d_i - S_i^*$.

The solution of the equation (23) will be

$$A_i^0 = \frac{D_i + \sqrt{l_n D_n D_i / l_i}}{a_i^*}; i = 1, 2, \dots, n-1; A_n^0 = \frac{D_n}{d_n}. \quad (24)$$

For each index value i the optimal solution is formed

$$\{A_i^{opt}\} = \begin{cases} A_i^0, & \text{if } A_i^- < A_i^0 < A_i^+ \\ A_i^-, & \text{if } A_i \leq A_i^0 \\ A_i^+, & \text{if } A_i \geq A_i^+ \end{cases}, i = 1, 2, \dots, n. \quad (25)$$

Comment. There is a minimum in problem (21), because

$$\frac{d^2 H}{dA_i^2} = A_i \left[l_i + D_n l_n D_i (a_i^*)^2 + D_n l_n D_i^2 a_i^* \right] \geq 0.$$

(Sufficient condition for the existence of an extremum).

4. Fuzzy formulation of the optimal design problem

Let \mathbf{A} be a subset, which is composed of fuzzy source data $\{x_i\}$; $i=1,2,\dots,k$, included in the set X , and which are specified by the modality quantifier "approximate". For example, "the applied load is approximately equal to 10kN", "the place of application of the load somewhere in the middle of the beam", "normal vibration frequency is about 10 Hz", "element length is close to 24 m.", etc. In this case $\mathbf{A} \subseteq X \subseteq U \subseteq R$, where U denotes a universal set included, for example, in the set of real numbers R .

For definiteness, let the following initial data be fuzzy: loads applied to node 4 and 5 (Fig. 1) and frequency ω_0 , so:

$$\mathbf{A}: \{\xi_1 = P_4; \xi_2 = P_5; \xi_3 = \omega_0\}.$$

So, in restriction $\sum_{i=1}^n \frac{D_i}{A_i} \leq \Delta_0$ of the problem (14) values D_i and Δ_0 are functions of fuzzy variables $\xi = (\xi_1, \xi_2, \xi_3)$. With account of this, the implementation of optimization problem (14) is carried out within the framework of the FST. Wherein V^{opt} and A^{opt} are also fuzzy sets.

5. Fuzzy modeling

Definition 1. Fuzzy modeling refers to the process of performing the following steps:

- operation of fuzzification of initial fuzzy values;
- carrying out calculations related to the solution of the problem of analysis or optimization with deterministic data of the level. Constructing a fuzzy result set;
- defuzzification of the obtained fuzzy results (expected value).

5.1. The first stage of modeling fuzzy statements about the data is performed using the basic concept of the theory of fuzzy sets - the membership function (MF) $\mu_A(x)$, which reflect each element x from subset $\mathbf{A} \subseteq X$ on the range of real numbers $[0,1]$, that is

$$\mu_A(x): X \rightarrow [0,1]. \quad (26)$$

It shows the level of belonging $\forall x \in X$ to the subset \mathbf{A} . Set X is the definitional domain of the function $\mu_A(x)$.

5.2. Application of α - level approach.

An important tool for FST is a α - level approach to the construction of MF. It is performed in the second stage of the simulation.

Definition 2. The α - level set of fuzzy set \mathbf{A} , given on a universal set U , is called a usual (determine) subset $\mathbf{A}_\alpha \subseteq X$, consisting of elements whose value of belonging to a fuzzy set \mathbf{A} no less than α , that is

$$\mathbf{A}_\alpha = \{x \in X \subseteq U \mid \mu_A(x) \geq \alpha\}.$$

Union of all subsets \mathbf{A}_α forms a fuzzy set \mathbf{A}

$$\mathbf{A} = \bigcup_{\alpha \in [0,1]} \alpha \mathbf{A}_\alpha. \quad (27)$$

For continuous functions $\mu_A(x)$ bound of subset \mathbf{A}_α

$$x^L(\alpha) = \inf \mathbf{A}_\alpha \text{ и } x^R(\alpha) = \sup \mathbf{A}_\alpha \quad (28)$$

determined by solving the equation

$$\mu_A(x) = \alpha; \alpha \in [0,1]. \quad (29)$$

The second stage of the modeling process is based on the application of the principle of generalization and α - level approach.

Let's consider the case when the transformation f is a function of many fuzzy variables.

If fuzzy independent values \tilde{x}_i given as $\tilde{x}_i = \bigcup_{\alpha \in [0,1]} [x_i^L(\alpha), x_i^R(\alpha)]$, then value

$\tilde{y} = f(\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_n)$ is fuzzy number $\tilde{y}_i = \bigcup_{\alpha \in [0,1]} [y_i^L(\alpha), y_i^R(\alpha)]$, where

$$\begin{aligned} y_i^L(\alpha) &= \inf(f(x_{1j}(\alpha), x_{2j}(\alpha), \dots, x_{nj}(\alpha))); \\ y_i^R(\alpha) &= \text{Sup}(f(x_{1j}(\alpha), x_{2j}(\alpha), \dots, x_{nj}(\alpha))). \end{aligned} \quad (30)$$

$$x_{ij}(\alpha) \in [x_i^L(\alpha), x_i^R(\alpha)];$$

$$x_{ij}(\alpha) = x_i^L(\alpha) + (x_i^R(\alpha) - x_i^L(\alpha))\xi_j; \quad \xi \in [0,1] = \text{random}, \quad (31)$$

$i = \overline{1, N}$, N - given number of α -levels; $j = 1, 2, \dots, M$; M - number of statistical tests.

According to this principle for each level $\alpha \in [0,1]$ the solution of optimization problems (30) is performed taking into account the requirement (31). Number of levels N is selected from the condition of ensuring the required accuracy.

The definition of a fuzzy number is the same as a fuzzy set. However, there are also additional requirements for the MF. They are normality, continuity and convexity.

So, the result of α - level approach to calculation $Y = f(x_i); i = \overline{1, n}$ is written in the form of a fuzzy set, that is:

$$Y^{fuz} = \sum_{i=1}^N \frac{\mu_i}{y_i^L} + \sum_{i=N+1}^{2N-1} \frac{\mu_i}{y_i^R}, \quad (32)$$

where i - level number; N - number of all levels; \sum - union sign;

$$\Delta\alpha = \frac{1}{(N-1)}; \mu_i = (i-1)\Delta\alpha; \mu_i = t\Delta\alpha; t = 2N-1-i.$$

5.3. Defuzzification stage. At this stage of the simulation, the problem of reflection of obtained fuzzy set Y^{fuz} into the single determine value $Y^{df} \in Y$ is performed. Such reflection is called defuzzification. In the work were used the following models to obtain Y^{df} :

1. Center average defuzzification

Here Y^{df} calculated by the formula [15]

$$Y^{df} = \sum_{i=1}^{2N-1} w_i Y_i^{fuz}, \quad w_i = \frac{\mu_i}{\sum_{i=1}^{2N-1} \mu_i}, \quad (33)$$

2. Expected value model (EVM)

The weights w_i , $i=1, 2N-1$ in the formula (33) determined by the following ratios proposed in [11]:

$$w_1 = \frac{1}{2}(\mu_1 + g_1 - g_2), \quad \text{for } i=1; \quad w_i = \frac{1}{2}(g_3 - g_4 + g_5 - g_6), \quad \text{for } 2 \leq i \leq r-1;$$

$$w_r = \frac{1}{2}(g_1 - g_7 + \mu_r), \quad w_0 = w_{2N-1} = 0, \quad (34)$$

where: $r = 2N - 1$, $g_1 = \max_{1 \leq j \leq r} \mu_j$; $g_2 = \max_{1 < j \leq r} \mu_j$; $g_3 = \max_{1 \leq j \leq i} \mu_j$; $g_4 = \max_{1 \leq j < i} \mu_j$;

$$g_5 = \max_{i \leq j \leq r} \mu_j; \quad g_6 = \max_{1 < j \leq r} \mu_j; \quad g_7 = \max_{i \leq j < r} \mu_j.$$

6. Numerical illustrations

6.1. An illustrative example of HRS optimal design.

6.1.1 Deterministic approach.

Let's consider the optimal design of the truss, the design scheme of which is shown in Fig. 1. The deterministic input data for the truss are: $h = 3m$, $L = 24m$, $l = 6m$, $E = 209GPa$, $g = 9.81m / sec^2$, $R_0 = 395MPa$, $\varphi = 0.722$

$P_1 = P_2 = P_3 = P_4 = P_5 = 10^4 kN$, $\omega = 12 hertz$, M - enough big value.

The efforts $N = (N_1, N_2, \dots, N_{18})$ are necessary to obtain an estimation of the cross sections A_i and coefficients D_i ; $i = \overline{1, 18}$. They are calculated by using the force matrix method [12]

$$N = BP,$$

where $B = \{b_{ij}\}$; $i = \overline{1, 18}$; $j = \overline{1, 5}$

For truss which consists of 18 elements (Fig. 1) the matrix B is like below. In the matrix there are notations like this: $b = l / L$; $c = ctg\alpha$; $s = 1 / \sin\alpha$; $\alpha = arctgH / l$.

The convergence of the implementation functional of the proposed procedure for dynamic programming is shown in Fig. 2. The optimal value of the volume is $V^{det} = 22588 cm^3$.

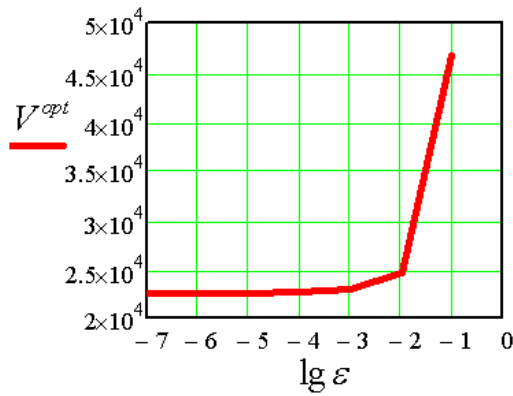


Fig. 2. The convergence dependence of the computational procedure of the DP in the deterministic problem of truss optimization according to the functional V^{opt} from the required accuracy ε

$$B = \left\{ \begin{array}{ccccc} -5bc & -3bc & -bc & -4bc & -2bc \\ -5bc & -3bc & -bc & -4bc & -2bc \\ (-15b+2)c & -9bc & -3bc & (-12b+1)c & -6bc \\ -3bc & -9bc & (-15b+2)c & -6bc & (-12b+1)c \\ -c & -3c & -5c & -2c & -4c \\ -c & -3c & -5c & -2c & -4c \\ 2bc & 6bc & (10b-1)c & 4bc & 8bc \\ 2bc & 6bc & (10b-1)c & 4bc & 8bc \\ (10b-1)c & 6bc & 2bc & 8bc & 4bc \\ (10b-1)c & 6bc & 2bc & 8bc & 4bc \\ 5bs & 3bs & bs & 4bs & 2bs \\ (-5b+1)s & -3bs & -bs & -4bs & -2bs \\ (5b-1)s & 3bs & bs & (4b-1)s & 2bs \\ bs & 3bs & (5b-1)s & 2bs & (4b-1)s \\ -bs & -3bs & -(5b-1)s & -2bs & -4bs \\ bs & 3bs & 5bs & 2bs & 4bs \\ -5b & -3b & -b & -4b & -2b \\ -b & -3b & -5b & -2b & -4b \end{array} \right\}$$

Let's present also the dependence of the displacement Δ from the natural frequency (Fig. 3)

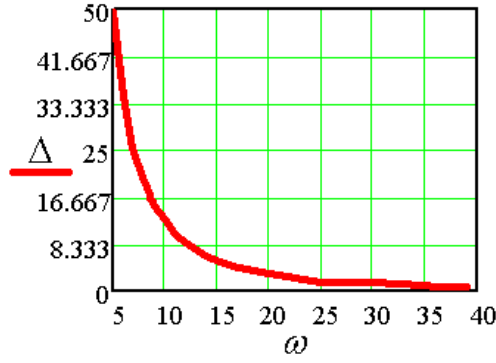


Fig. 3. The dependence of the displacement Δ from the natural frequency ω

From this it follows that an increase in the frequency of natural oscillations leads to a decrease in the displacement of the characteristic unit of the truss.

6.1.2 Fuzzy approach

Let some load acting on the farm be fuzzy, for example $\tilde{P}_4 = \tilde{P}_5 = \tilde{10} \cdot 10^3 \text{ kN}$. The natural frequency is also fuzzy and equal to $\omega_0 = \tilde{12} \text{ hertz}$.

For fuzzy numbers $\tilde{P}_i(a, m, b)$; $i = 4, 5$ and $\tilde{\omega}_0(a, m, b)$ the triangular membership function is taken:

$$\mu(x) = \begin{cases} \frac{x - m}{m - a}, & a \leq x < m \\ \frac{b - x}{b - m}, & m < x \leq b \\ 1, & x = m \\ 0 & \end{cases} \quad (35)$$

The solution of equation (29) for function (35) gives

$$x_{P_i}^L(\alpha) = m_{P_i} \alpha + (1 - \alpha) a_{P_i}; \quad x_{P_i}^R(\alpha) = m_{P_i} \alpha + (1 - \alpha) b_{P_i};$$

$$x_{\omega}^L(\alpha) = m_{\omega} \alpha + (1 - \alpha) a_{\omega}; \quad x_{\omega}^R(\alpha) = m_{\omega} \alpha + (1 - \alpha) b_{\omega},$$

where $i = 4, 5$; $\alpha \in [0, 1]$; a_{P_i} , b_{P_i} , a_{ω} , b_{ω} are, respectively, the boundaries of the intervals of fuzzy values \tilde{P}_4 , \tilde{P}_5 и $\tilde{\omega}_0$; m_{P_4} , m_{P_5} , m_{ω} are their modal values.

Let's enter into consideration the coefficient k of fuzziness in the initial data as a part in percentage (%) of modal values m_{P_4} , m_{P_5} , m_{ω} . Then

$$a_{P_i} = k(1 - 0.01m_{P_i}); \quad b_{P_i} = k(1 + 0.01m_{P_i}); \quad i = 4, 5;$$

$$a_{\omega} = k(1 - 0.01m_{\omega}); \quad b_{\omega} = k(1 + 0.01m_{\omega}); \quad (36)$$

If set, for example, coefficient is equal $k = 20\%$ and number of α -levels is equal $N = 10$, in accordance with the described DP procedure, we obtain the value of the optimal volume V^{fuz} , written in the form of a fuzzy set with the membership function shown in Fig. 4

$$V^{fuz} = \frac{0}{13333} + \frac{0.1}{14051} + \frac{0.2}{14840} + \frac{0.3}{15700} + \frac{0.4}{16589} + \frac{0.5}{17509} + \frac{0.6}{18461} + \frac{0.7}{119444} +$$

$$+ \frac{0.8}{20459} + \frac{0.9}{21507} + \frac{1}{22588} + \frac{0.9}{23702} + \frac{0.8}{24851} + \frac{0.7}{26034} + \frac{0.6}{27252} + \frac{0.5}{28505} +$$

$$\frac{0.4}{29794} + \frac{0.3}{31120} + \frac{0.2}{32482} + \frac{0.1}{33881} + \frac{0}{35318}.$$

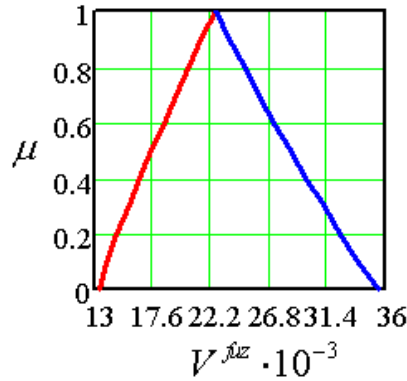


Fig.4. The membership function of a fuzzy value $V^{fuz} \cdot 10^{-3}, \text{cm}^3$

The result of defuzzification of a fuzzy number V^{fuz} describe the table 1.

Table 1

Center method	$V^{df} = 22865 \text{cm}^3$	1.23%
Method [11]	$V^{df} = 23068 \text{cm}^3$	2.13%

As can be seen from this table, the discrepancy from the solution V^{det} when using the method of centers more by 1% than when using the approach [11].

Performing coefficient k impact analysis of fuzziness of values $\tilde{P}_4, \tilde{P}_5, \tilde{\omega}_0$ on the volume of the designed truss, we get the result given in table. 2

Table 2

"Tolerance" η of the system from k coefficient

$k, \%$	V^{df}, cm^3	$\eta, \%$
20	22865	1.23
15	22744	0.7
10	22657	0.31
5	22605	0.08
1	22588	0

Here $\eta = \frac{V^{df} - V^{\text{det}}}{V^{\text{det}}} \cdot 100, (\%)$ – расхождение полученных V^{df} от значений V^{det} .

The influence of data coefficient ($k, \%$) and natural frequency ω on ($\eta, \%$) is given in the Fig. 5.

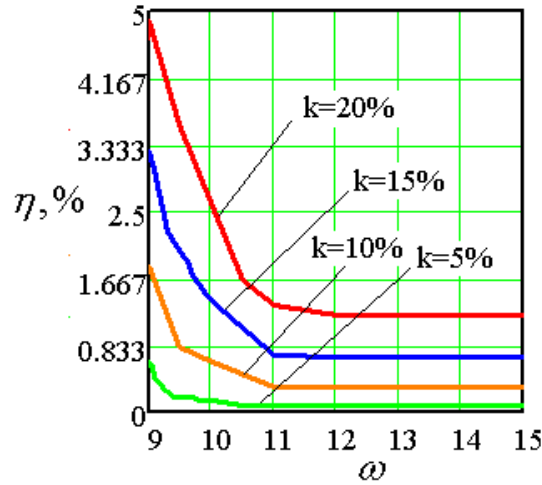


Рис. 5. Blur result from the coefficient of fuzziness and natural frequency

Maximum blur η in condition of fuzzy description of the data, as expected, arises when natural frequency is equal $\omega = 9 \text{ hertz}$ or corresponding displacement is equal $y^{\max} = 16 \text{ cm}$. These data are shown in the table 3.

Table 3

The dependence of the blurriness of the result from a fuzzy data setting

$k, \%$	$\eta, \%$
20	4.87
15	3.25
10	1.81
5	0.64

6.2 Fuzzy-random uncertainty

The fuzzy modeling procedure is applicable to the optimal truss design problem (Fig. 1), when the loads are P_4 and P_5 attached randomly, and their values are fuzzy with a triangular membership function. Finding the minimum volume of the structure, taking into account the constraints of strength and vibrations, we perform on the basis of fuzzy-random modeling [11].

Denote by B_1 the event when node 4 is loaded with \tilde{P}_4 given in a fuzzy way with probability p_1 ;

Denote by B_2 event when node 5 is loaded with \tilde{P}_5 given in a fuzzy way with probability p_2 . Let the fuzzy values $\tilde{P}_4(a_{P_4}, m_{P_4}, b_{P_4})$ and $\tilde{P}_5(a_{P_5}, m_{P_5}, b_{P_5})$ described by a triangular membership function. The model of the optimization problem remains the same as in (14).

Consider the joint law of probability distribution of events B_1 and B_2 :

1. Event $C_1 = B_1 B_2$ means that in the nodes 4 and 5 are applied load, respectively, equal \tilde{P}_4 and \tilde{P}_5 with probability $q_1 = p_1 p_2$.

2. Event $C_2 = \overline{B_1} B_2$ means that node 4 has an applied load \widetilde{P}_4 , and in node 5 there is no load, so $\widetilde{P}_5 = 0$. The corresponding probability of such an event is $q_2 = p_1(1 - p_2)$.

3. Event $C_3 = \overline{B_1} \overline{B_2}$ means that load \widetilde{P}_5 is applied to node 5, and in node 4 there is no load, so $\widetilde{P}_4 = 0$. The corresponding probability of such an event is $q_3 = p_2(1 - p_1)$;

4. Event $C_4 = \overline{B_1} \overline{B_2}$ means that nodes 4 and 5 have no load, so $\widetilde{P}_4 = 0$, $\widetilde{P}_5 = 0$. It will happen with probability $q_4 = (1 - p_1)(1 - p_2)$.

According to the procedure proposed above, we calculate V^{df} for each combination of loads.

We will perform the derandomization of the obtained results in accordance with the definition of the expectation value, that is

$$V^{dr} = \sum_{i=1}^4 q_i V_i^{df} .$$

With the following numeric data as: $P_1 = P_2 = P_3 = 10^4 kN$, $\widetilde{P}_4 = 10kN$, $\widetilde{P}_5 = 10^4 kN$, $g = 981 cm / sec^2$, $\omega = 12 hertz$, $p_1 = 0.7$, $p_2 = 0.8$ and coefficient $k = 10\%$ get the results shown in table 4.

Comment. Values V_i^{df} ; $i = \overline{1,4}$ are obtained in accordance with the requirement of strength of elements $R_0 = 395 \cdot 10^2 MPa$, $\varphi = 0.722$ and the effects of the "worst" deterministic load as: $P_i = 10^4 \kappa H$, $P_4 = P_5 = (1 + \frac{k}{100}) 10^4 \kappa H$.

Table 4

1	Event	C_1	C_2	C_3	C_4	
2	Probability q_i	0.56	0,14	0,24	0,06	$\sum q_i = 1$
3	Volume V_i^{df}	22588	18023	18772	16029	

The operation of derandomization gives the value of the volume $V^{dr} = 20640 cm^3$. From table 4 it can be seen that if the absence of a load on the upper chord of the truss then $V^{df} < V^{dr}$ on 22%, and if the loads present on the upper chord then $V^{df} > V^{dr}$ on 9.4%. It should be noted that the value of the desired volume is within $16029 < V^{dr} < 22588$, that is, between solutions at the lowest and highest loads.

6.3. The problem of fuzzy analysis

Consider the problem of determining of the bending moment M_C under the axial force applied at point C on distance c from point B for elastic beam with two clamped ends (Fig. 6)

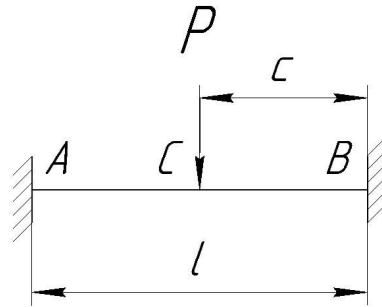


Fig. 6. Design scheme of the beam

In accordance with [14], the bending moment at point C is determined by the expression

$$M_C = \frac{2P[c(l-c)]^2}{l^3} = f(c, P)$$

Information situation 1.

Let the value of the load P and the place of its application is described in a fuzzy way: value P "close to 20 kN", and point C is located "near the middle of the beam". For this case it is necessary to obtain the value M_C of bending moment under the point of force application. In the notation (1) we can write the problem as $f : X \rightarrow M_C; x \in X; x = (\xi_1, \xi_2); \xi_1 = \tilde{P}, \xi_2 = \tilde{c}$.

Without breaking the common approach, we assume that x_1 and x_2 are set by the triangular type membership function (35) at the fuzzification stage. Numbers $(a_p, m_p, b_p), (a_c, m_c, b_c)$ are carriers of fuzzy numbers \tilde{P} and \tilde{c} , so a_p, b_p, a_c, b_c set interval limits, where $\mu(x) \geq 0; m_p, m_c$ are modal values.

The next stage begins with the function values f calculation of two fuzzy variables x_1 and x_2 based on the principle of generalization and α -level approach. The numerical experiment will perform when $l = 6m, m_p = 20kN, m_c = 3m$. As result we obtained the fuzzy numbers of moment M_C (Fig. 7) and and defuzzified values of the moment M_C for different degrees of fuzziness. The data is shown in fig. 8 and in table. 5.

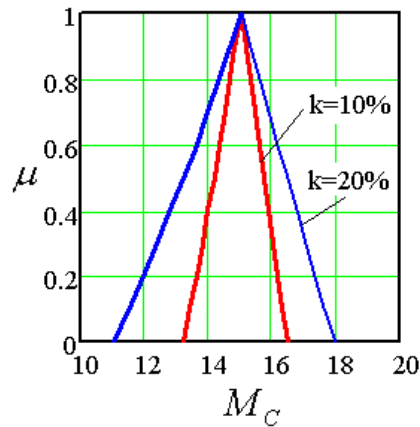


Fig. 7. Fuzzy numbers of the moment M_C for such degrees of fuzziness as 10% and 20%

Table 5

Information situation when " P is approximately equal $20kN$ " and " c is equal to about 3 meters"

$k, \%$	$M^{df}, N \cdot m$	$\eta, \%$
0	15	0
5	14.992	0.06
8	14.981	0.13
10	14.972	0.19
12	14.961	0.26
15	14.942	0.39
20	14.904	0.64
25	14.858	0.95
30	14.805	1.3

Here in the table. 5 value $\eta, \%$ is a deviation (degree of tolerance) of the defuzzified value M^{df} from deterministic value M^{det} .

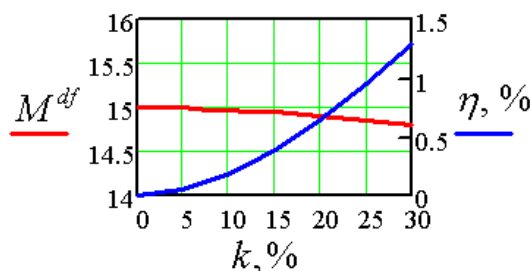


Fig. 8. Dependence of the defuzzified value M^{df} and its deviation versus M^{det} from degree of blurring.

The result is:

- MF of the moment M_C (fig. 7) for $k = 10\%$ and $k = 20\%$;
- dependence of defuzzified value M_C from the degree of blurriness of the initial fuzzy data (Fig. 8);

- the dependence of the deviation degree η of the result from the value of the fuzziness of the initial data (tolerance);
- with increasing the value k the bending moment decreases.

Information situation 2

The informational situation is characterized, unlike the first one, in that the description of the point of load application remains the same as above, and the value of the load is described differently: “the value of the concentrated force is slightly greater than 20 kN”. The fuzzification of such a description can be accomplished by introducing the definition of a semi-fuzzy number whose membership function is

$$\mu(x) = \begin{cases} 1, & x = m \\ \frac{b-x}{b-m}, & m < x \leq b \\ 0. & \end{cases}$$

The results are presented in Table. 6 and Fig. 9.

Table 6

The results of the analysis, when “P is slightly larger than 20kN ”, and “c is about 3 meters ”

$k, \%$	$M^{df}, N \cdot m$	$\eta, \%$
0.5	15.012	0.08
1	15.024	0.16
5	15.116	0.78
10	15.220	1.47
15	15.312	2.08
20	15.392	2.61
25	15.46	3.07
30	15.516	3.44

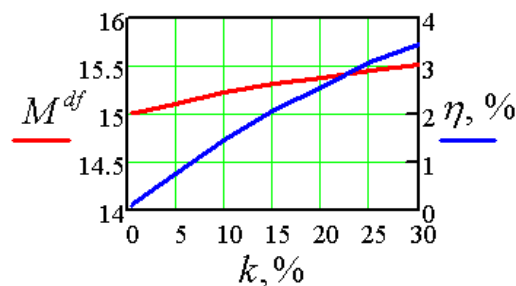


Fig. 9. The defuzzified value M^{df} and its deviation versus the M^{det} form the degree of blurring

As can be seen, an increase in the degree of data blurring k leads to a slight increase of the value of the bending moment M^{df} and coefficient η .

Conclusions.

1. A computational approach is presented to solve problems of analysis and optimization of structures in case when the initial data are described in a fuzzy way.
2. The development of the approach is based on the application of the rules of the theory of fuzzy sets L. Zadeh and fuzzy modeling, which is illustrated in the problem of structural mechanics and optimal design.
3. Information on the impact of the degree of fuzziness of the initial data on the design result is given. Comparison of the result with the solution with certain data is obtained.
4. A computational algorithm of step by step approach, based on the use of the dynamic programming method, is proposed for the truss optimization problem. Its convergence is shown.

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