УДК 519.6:628.35
DOI: 10.30838/UJCEA.2312.270425.165.1185

MODELING THE DYNAMICS OF GROUNDWATER IN APPLIED PROBLEMS

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Abstract. ***Problem statement.*** Forecasting and analyzing groundwater dynamics under anthropogenic impact on aquifers is a pressing issue. This task arises when designing drainage systems for flooded areas, designing water intakes from underground sources, the impact of waste disposal sites on the movement of groundwater, and water leakage from water supply and sewage systems. Anthropogenic impact can affect both groundwater (unconfined aquifers) and confined aquifers. Technogenic impact can occur through a system of wells, leakage of effluents from landfills, infiltration of precipitation, and irrational irrigation. Solving this type of problem requires the use of multifactorial filtration models. ***The purpose of the article***. To conduct a comprehensive analysis of mathematical models in groundwater dynamics problems and to identify their advantages and disadvantages. ***Conclusions.*** The analysis showed that the modern approach to solving groundwater dynamics problems is to use multidimensional filtration equations that take into account changes in the groundwater level (or hydrodynamic pressure in the confined aquifer) over time. These modeling equations take into account water infiltration into underground horizons. The boundary conditions of the modeling filtration equations that ensure the correct formulation of the boundary problem are considered. It is shown that the formulation of specific groundwater dynamics problems (drainage, operation of water intake wells, etc.) is implemented in models by setting "internal" boundary conditions. Applied problems are analyzed where models of unconfined and confined groundwater flow can be used.

Keywords: *groundwater dynamics; drainage; underground waste disposal; water supply from underground source; mathematical modeling*

МОДЕЛЮВАННЯ ДИНАМІКИ ПІДЗЕМНИХ ВОД В ПРИКЛАДНИХ ЗАДАЧАХ

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Анотація. ***Постановка проблеми.*** Актуальною проблемою е прогнозування та аналіз динаміки підземних вод при антропогенному впливу на водоносні горизонти. Дана задача виникає при проектуванні систем дренажу підтоплених територій, проектуванні водозаборів з підземних джерел, впливу полігонів відходів на режим руху підземних вод, витоку води с систем водопостачання та каналізації. Антропогенне навантаження може бути як на ґрунтові водо (безнапірний водоносний горизонт) так і на напірні водоносні горизонти. Техногенний вплив може здійснюватися крізь систему свердловин, витоку стоків з полігонів, інфільтрації опадів, нераціонального зрошування. Рішення такого класу задач потребує використання багатофакторних моделей фільтрації. ***Мета роботи.*** провести комплексний аналіз математичних моделей в задачах динаміки підземних вод та визначити їх переваги та недоліки. ***Висновки.*** Проведений аналіз показав, що сучасним підходом при рішенні задач динаміки підземних вод є використання багатовимірних рівнянь фільтрації, що враховують зміну рівня ґрунтових вод (або гідродинамічного напору в напірному водоносному горизонті) з часом. Дані моделюючи рівняння враховують інфільтрацію води в підземні горизонти. Розглянуто крайові умови моделюючих рівнянь фільтрації, що забезпечують коректну постановку крайової задачі. Показано, що формулювання конкретних задач динаміки підземних вод (дренаж, робота свердловин системи водозабору тощо) реалізуються в моделях шляхом задання «внутрішніх» граничних умов. Проаналізовані прикладні задачі, де можуть бути використані моделі руху безнапірного та напірного підземного потоку.

Ключові слова: *динаміка підземних вод; дренаж; підземна утилізація відходів; водопостачання з підземного джерела; математичне моделювання*

Introduction. An important practical task is the study of groundwater dynamics under various types of technogenic load. For practice, it is important to know:

1) the change in groundwater regime during the operation of drainage systems in flooded areas;

2) the change in groundwater dynamics during the underground disposal of industrial waste and the filtration of water from landfills into the underground aquifer;

3) the change in the depth of the aquifer, hydraulic pressure during the operation of wells in water supply systems.

It should be emphasized that due to the specifics of the tasks of this class (the impossibility of seeing the movement of groundwater), an important task is the development of mathematical models. Mathematical models can be used to predict changes in the dynamics of groundwater when operating drainage systems, constructing hydraulic structures, designing water supply systems, etc. For practice, it is very important to have quick-calculation mathematical models that allow for the rapid determination of the depth of groundwater, the amount of water, and the hydrodynamic pressure in the aquifer under anthropogenic influence. It should also be noted that the method of physical modeling also plays an important role in the field of groundwater dynamics research: based on the data from physical experiments, a number of parameters (coefficient of filtration, porosity, etc.) are determined, which are necessary for conducting computational experiments.

Analysis of publications. The problem of modeling groundwater dynamics is discussed in the scientific works of many authors [1–9]. These works consider the use of empirical models, analytical and numerical models in geomigration tasks. However, for practice, it is very important to have a systematic material regarding existing models of groundwater dynamics, their areas of application, advantages, and disadvantages.

The purpose of the article is to conduct a comprehensive analysis of mathematical models in groundwater dynamics tasks and to determine their advantages and disadvantages.

Research results. Conducting physical experiments in the field of groundwater dynamics requires significant time. Therefore, considerable attention is paid to the development of mathematical modeling methods. These methods have obvious advantages:

1. Significantly lower economic costs compared to physical experiments.

2. The ability to quickly obtain the necessary forecast data regarding the 'behavior' of the object.

3. The availability of well-developed tools for mathematical modeling of filtration and geomigration processes.

4. When conducting physical experiments, the researcher cannot 'look in' and see how the pollution zone forms in the underground flow (unlike, for example, the formation of pollution areas from industrial discharges), but the use of mathematical modeling allows for the 'visualization' of the shape and intensity of pollution areas in the underground flow.

5. Mathematical modeling allows for the rapid response of groundwater to changes in various parameters: for example, to predict how the level of groundwater pollution will change with an increase in well discharge, etc. This is very important, especially during the design phase.

Within the scientific direction of 'mathematical modeling', the following types of models can be distinguished:

1. Empirical models. This is a group of models that are mainly used for quick solutions to filtration and mass transfer problems. These models take the form of simple algebraic formulas. When constructing empirical models, data from field observations and results from physical experiments are used.

Advantages of empirical models:

1) simplicity of computational dependencies (the computational formulas do not contain special functions, such as Bessel functions, no integrals, etc.).

2) there is no need to use a computer for applying the models.

3) high-qualified personnel are not required for the practical application of models of this class.

Disadvantages of empirical models:

1) models can only be applied to the conditions under which the experimental data were obtained, on the basis of which the empirical model was created; applying these models to other conditions requires scientific justification.

2) models use a limited number of physical parameters for calculations, so these models do not allow for a sufficiently complete picture of the geomigration process (for example, they do not allow for determining the intensity and shape of the pollution area in the underground flow).

3) models may contain parameters that have no physical meaning (how can these parameters be verified then?).

It should be emphasized that empirical models are used in practical calculations to determine some parameters of geomigration (for example, the dispersion coefficient).

Within the framework of modern requirements for the quality of predictive information, the use of empirical models to solve multifactorial geomigration problems is impractical.

2. Analytical models. Analytical models in filtration and mass transfer problems are a powerful research tool. Analytical models are computational dependencies that are exact solutions to the equations of filtration or mass transfer. Various mathematical integration methods are used to obtain such computational dependencies: the method of conformal mappings, the Fourier method, the method of potential flows, etc.

Based on analytical models, a wide range of filtration problems has been solved that arise in the design of hydraulic structures, drainage systems, anthropogenic contamination of groundwater, soil salinization, etc.

Advantages of analytical models:

1) since these models are the “exact” solution to the geomigration problem or the filtration problem, they are an “undeniable” mathematical result.

2) an analytical model can be used to verify numerical models.

3) for a number of cases (for example, modeling the operation of a well), the models allow obtaining a satisfactory solution to a real applied problem.

4) models can be used to solve the "inverse" problem – to determine, for example, the intensity of the source of anthropogenic pollution based on experimental data.

5) models can be used to study the "importance" of the influence of a particular physical parameter on the process of filtration and geomigration.

6) models can be applied to solve complex real problems in the absence of sufficient input information at the beginning of the research.

7) obtaining a solution to a predictive problem based on an analytical model, in many cases, does not require the use of a computer.

Disadvantages of analytical models:

1) the computational dependency may contain special functions and integrals, which significantly complicates the practical application of the model.

2) a certain level of user qualification is required.

3) the area of application of analytical models is significantly limited. This is due to the fact that an analytical solution to filtration and geomigration problems can only be obtained by using significant simplifications in the formulation of the problem, for example, by considering only one-dimensional flow. This significantly narrows the range of practical applications of models of this class.

3. Numerical models. Numerical models [10−12] are created by applying various approximate (numerical) methods for solving boundary problems of filtration and geomigration. When solving filtration and geomigration problems, the following numerical integration methods for modeling equations are most commonly used:

1) finite difference methods;

2) finite element method;

3) complex boundary element method (numerical-analytical method).

The use of numerical models in practice is referred to as "numerical modeling".

A feature of numerical models is that their application necessarily requires a computer, that is, to apply a numerical model, it is necessary to develop computer code (a program for calculations on a computer). This code is a tool in the scientific research conducted. Therefore, users of numerical models are divided into the following groups:

1. Users who cannot develop numerical models and create computer codes based on them, therefore they use ready-made software products, that is, commercial codes (for example, COMSOL code).

2. Users who use commercial codes but can independently develop a computer program that allows them to "refine" the result obtained using the commercial code for a specific situation (for example, a self-developed code that additionally calculates the risk of suffusion in a specific area of groundwater movement).

3. Users who independently develop numerical models and create computer codes based on them (author codes).

In the practice of solving geomigration problems, commercial codes such as MODFLOW and COMSOL are widely used, which implement numerical models of filtration and geomigration.

Advantages of numerical models:

1. The ability to take into account a significant number of physical factors that influence the formation of pollution halos during modeling.

2. A wide working range (the ability to model the dynamics of groundwater and geomigration processes in pressurized groundwater flows, unpressurized, etc.).

3. Significantly less time to obtain predictive data compared to field studies.

4. The model can be used to solve the 'inverse' problem – to determine, for example, the intensity of a source of anthropogenic pollution based on experimental data.

Disadvantages of numerical models and commercial codes:

1. When using commercial codes, the user must have a license to use the commercial code in scientific research (i. e., the user must undergo official training).

2. Commercial codes are oriented towards solving a specific class of problems within a particular mathematical model, boundary conditions, and grid. The user cannot independently configure the commercial code to solve a different class of problems.

3. In areas with a large gradient of functions, commercial codes may 'fail', meaning that a solution cannot be obtained.

4. High cost of licensed commercial codes.

5. High qualification of the user is required.

It should be noted that the application of numerical modeling is currently a major trend in solving geomigration problems.

For analyzing the dynamics of groundwater (unpressurized mode), the following equation is used:

$$\mu \frac{\partial h}{\partial t} = kh_m \left(\frac{\partial h^2}{\partial x^2} + \frac{\partial h^2}{\partial y^2} \right) + \\ + \sum W_i \delta(\mathrm{x} - \mathrm{x}_i)\delta(\mathrm{y} - \mathrm{y}_i), \quad (1)$$

where h – depth of flow; k – coefficient of filtration; μ – water yield; h_m – average depth of flow; W_i – intensity of mineralized water supply to the i-th well; x_i, y_i – coordinates of the well.

Modeling the dynamics of underground flow is carried out in an area that has a rectangular shape. For equation (1) of the dynamics of unpressurized groundwater movement, the following boundary conditions are implemented:

1. At $t = 0$, the depth of the underground flow in the study area is set.

2. At the boundary where the flow enters the study area, the depth value $h = h_1$ is set, where h_1 is the known depth of flow.

3. At the boundary where the flow exits the study area, the depth value $h = h_2$ is set, where h_2 is the known depth of flow.

4. On the lateral sides of the computational area, a boundary condition is implemented, where n is the unit vector of the outward normal to the boundary.

The components of the underground flow velocity are determined based on Darcy's law:

$$u = -k \frac{\partial h}{\partial x}, v = -k \frac{\partial h}{\partial y}, \quad (2)$$

where k – coefficient of filtration.

Numerical methods are used for the numerical solution of the modeling equation (1).

It should be noted that equation (1) can be used to solve the following problems:

1. Determining the depth of flow when the drainage system is operating; in this case, where the well is located that provides groundwater pumping, the depth corresponding to the specific flow rate of the well is set;

Another approach is to set the flow rate W at the location of the well.

2. Analyzing the dynamics of groundwater during the infiltration of contaminated effluents from waste landfills into the aquifer;

3. Studying the regime of groundwater during underground waste disposal.

For analyzing the dynamics of pressurized groundwater, the following equation is used:

$$\frac{1}{a_n} \frac{\partial H}{\partial t} = \left(\frac{\partial H^2}{\partial x^2} + \frac{\partial H^2}{\partial y^2} \right) + \\ + \sum W_i \delta(\mathrm{x} - \mathrm{x}_i)\delta(\mathrm{y} - \mathrm{y}_i), \quad (3)$$

where a_n – the piezoconductivity coefficient (according to V. Shchelkachov); H – hydrodynamic pressure; W_i – intensity of mineralized water supply to the i-th well; x_i, y_i – coordinates of the well.

Modeling the dynamics of pressurized underground flow is carried out in an area that has a rectangular shape. For equation (3) of the dynamics of pressurized groundwater movement, the following boundary conditions are implemented:

1. At $t = 0$, the hydrodynamic pressure in the study area is set.

2. At the boundary where the flow enters the study area, the value $H = H_1$ is set, where H_1 is the known hydrodynamic pressure.

3. At the boundary where the flow exits the study area, the value $H = H_2$ is set, where H_2 is the known hydrodynamic pressure.

4. On the lateral sides of the computational area, a boundary condition is implemented, where n is the unit vector of the outward normal to the boundary.

The components of the velocity vector of the underground flow are calculated based on Darcy's law.

Numerical methods are used to solve the modeling equation (3). For example, numerical integration of the modeling equation (3) is implemented in the commercial package MODFLOW.

It should be noted that equation (3) can be used in solving the following problems:

1. Determining the hydrodynamic pressure during the operation of the water intake; in this case, where the well is located, the piezometric pressure is set, or the well discharge W is specified.

2. Analysis of groundwater dynamics during underground waste disposal. In this case, where the well is located through which the discharge is made, the piezometric pressure is set.

Conclusions

1. An analysis of fundamental models of groundwater dynamics has been carried out; their advantages and disadvantages have been identified.

2. Multifactor equations of groundwater movement and boundary conditions of the modeling equations of filtration are presented, ensuring the correct formulation of the boundary problem.

3. Applied problems have been analyzed where models of unconfined and confined groundwater flow can be used.

4. This scientific direction should be further developed in the field of developing fast-calculating numerical models for analyzing groundwater dynamics.

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Надійшла до редакції: 12.04.2025.