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TOWARDS THE PROBLEM OF SYNTHESIS OF DECISIONS UNDER FUZZY CONDITIONS: AGREEMENT OF INDIVIDUAL EXPERT ASSESSMENTS

The essence of the method of expert evaluations is a logical-intuitive analysis of the internal and external environment of the management object, the development of alternatives and a quantitative assessment of their quality. The generalized opinion of experts serves as a basis for choosing an adequate decision option. The complex use of intuitive and logical thinking, as well as the formal processing of experts judgments allows to obtain indicators of the quality of alternatives in a multi-criteria decision-making task. One of the common methods of processing expert information is the method of paired comparisons by T. Saati. Each expert submits conclusions in the form of a square matrix, the elements of which are formed according to the scale of the relative importance of the alternative. The consistency of the matrix of pairwise comparisons means that the relative weight coefficients of the alternatives retain their value, regardless of how the comparison is performed. Given that pairwise comparison matrices are built on the basis of human judgment and assumptions, some degree of inconsistency between its elements is to be expected. Such inconsistency is not critical, if it does not go beyond the defined limits. The article considers the method of harmonizing individual assessments of experts in the problem of synthesis of a decision under unclear conditions.

Keywords: alternative, expert assessments, consistency, matrix of paired comparisons, criterion.

JEL classification: C44, C 46, C65

ПРОБЛЕМИ СИНТЕЗУ РІШЕННЯ ЗА НЕЧІТКИХ УМОВ: УЗГОДЖЕННЯ ІНДИВІДУАЛЬНИХ ЕКСПЕРТНИХ ОЦІНОК

Більшість аналітичних задач вибору та порівняння альтернатив вимагають використання незалежних суджень, що може бути досягнуто лише залученням експертів. В силу суб'єктивності індивідуальних оцінок, експерти можуть формувати різні висновки, іноді рішення отримані від експертів протирічають між собою. В цьому випадку необхідно узгодити рішення будь-яким прийнятним способом і лише після цього інформація, отримана від експертів, вважається прийнятною для використання. Сутьність методу експертних оцінок полягає в логіко-інтуїтивному аналізі внутрішнього і зовнішнього середовища об'єкта управління, розробці альтернатив і кількісній оцінці їх якості. Узагальнена думка експертів служить підставою для здійснення вибору адекватного варіанта рішення. Комплексне використання інтуїтивного і логічного мислення, а також формальної обробки суджень експертів дозволяє отримати показники якості альтернатив в багатокритеріальній задачі прийняття рішення. Одним із поширених методів обробки експертної інформації є метод парних порівнянь Т. Сааті. Кожен експерт подає висновки у формі квадратної матриці, елементи якої формуються за шкалою відносної значимості альтернативи. Узгодженість матриці парних порівнянь означає, що відносні вагові коефіцієнти альтернатив, зберігають своє значення, незалежно від того як виконується порівняння. Узгодженість матриці парних порівнянь може не досягатися, оскільки судження експерта не можна оцінити числовим еквівалентом. Неоднорідність матриці парних порівнянь може бути викликана як особистими уподобаннями експерта, так і ступенем невизначеності об'єкта оцінки. Враховуючи, що матриці парних порівнянь будуються на основі людських суджень та припущень, очікуваним буде наявність деякої міри неузгодженості між її елементами. Така неузгодженість є не критичною, якщо не виходить за визначені межі. Використання оцінок альтернатив, отриманих на основі висновків експертів, є способом врахування персонального знання та досвіду фахівців в умовах часткової невизначеності та суб'єктивності інформації, коли застосування детермінованих аналітичних алгоритмів обґрунтування рішень не можливе, або не доцільне. У статті розглядається метод узгодження індивідуальних оцінок експертів в задачі синтезу рішення за нечітких умов.

Ключові слова: альтернатива, експертні оцінки, узгодженість, матриця парних порівнянь, критерій.

Formulating a problem. Modern problems in the theory and practice of management are related to the progressive dynamics of the environment external to the object of management. The speed of changes in the economic, political and technological environment forces decisions to be made more quickly, in an adaptive mode. Global challenges such as pandemics, climate change, and geopolitical instability create a high level of uncertainty

in both decision criteria and alternatives. Huge amounts of data generated by modern information technologies create an excess of unstructured information (information noise), which users have to analyze in the process of generating a decision. The enumerated, but not complete, list of factors increases the risk of erroneous decisions – the choice of an inadequate management reality. The main difficulties of algorithmization arise when the parameters

of the control model are uncertain, but their influence on the result is simultaneous and significant. Parametric uncertainty changes the structure and analytical form of the model, leads to the need to study the intervals of parameter values. In formal models, deterministic methods are most often used, in which the inaccuracy of certain parameters during calculations is practically not taken into account or, taking into account certain limitations and assumptions, inaccurate parameters are replaced by expert estimates and weighted average values. The resulting violations of structural equations, balance ratios, and target functions lead to the need for parameter variations, which does not always ensure obtaining an acceptable result. Such situations can arise both as a result of insufficient study of the objects of management, and due to the participation in the management of a person or a group of persons as subjects of decision-making, when a significant part of the information is presented only in the linguistic form of conclusions or wishes of experts. But traditional mathematics does not have tools with which it would be possible to accurately reflect the vagueness of the ideas of experts. Quantitative methods of system analysis are inherently not very suitable and ineffective for such decision-making problems. This is determined by the so-called system principle of incompatibility: the more complex the system, the less accurate and meaningful the experts judgments about its behavior. For systems whose complexity exceeds a certain threshold level, accuracy and practicality become almost exceptional. Precisely in this sense, accurate quantitative analysis, traditional methods of research of real economic, social and other systems related to human participation, is of no practical importance. They are unable to grasp the vagueness of human thinking and behavior. The "incompatibility" of complex systems can be smoothed out if we assume that the elements of human thinking are not numbers, but elements of some vague sets or classes of objects for which the transition between classes is defined (from "belonging" to "non-belonging"). Then the result of the formalization of uncertainty of various nature will be fuzzy sets and relations, and the synthesis of the solution is carried out in fuzzy conditions. Complex use of intuitive and logical thinking, as well as vague methods of processing experts judgments, allows to increase the adequacy of decision selection.

Analysis of recent research and publications.

The essence of the method of expert evaluations consists in the logical and intuitive analysis of the internal and external environment of the organization, the development of alternatives and the quantitative assessment of their quality [1; 2]. The generalized opinion of experts serves as a basis for making a choice. At the same time, experts act as "measurers of information". The choice of forms and methods of conducting expert surveys, approaches to processing the survey results depends on the problem and conditions of the examination [3]. The feasibility of using expert assessments is determined by the form of presentation and the degree of formalization of input information within the scope of the management task [4]. The entire set of data received from experts is the result of measurement, or more precisely – a comparison of situations, objects, alternatives according to selected indicators or signs. At the same time, the comparison procedure is based on the existence of a fundamental possibility of establishing

certain relations of order between the characteristics of processes and objects. The simplest among order relations are binary ones, which allow you to obtain pairwise comparisons of decision options [4; 5]. The legitimacy of the use of expert assessments is based on the hypothesis: the unknown characteristic of the phenomenon under study is a random variable, and the individual assessment of an expert specialist of this characteristic is interpreted as a separate implementation of a random variable [6; 7]. Hypothetically, the mathematical expectation of the studied characteristic is within the range of expert group assessments, i.e., group expert judgment can be considered reliable. But, for a certain class of little-researched problems, this hypothesis is not acceptable, because in this case the opinion of one expert may turn out to be the most significant, and it will be lost during processing. Individual methods of expert assessment avoid this problem and allow the expert to express his opinion independently of others [8; 9]. However, at the same time, it is necessary to agree the opinions of all experts regarding the investigated problem.

Formulation of goals. The object of the study is the method of harmonizing individual assessments of experts within the framework of the problem of synthesis of a fuzzy solution.

Main material presentation. Most analytical tasks of choosing and comparing alternatives require the use of independent judgment, which can only be achieved by involving experts. Individual methods of expert evaluation allow the expert to express his independent opinion. However, at the same time, it is necessary to agree on the opinions of all experts. These judgments should not be contradictory, that means, the assessments given by them should be checked for consistency and only after that the information received from the experts is considered acceptable for decision-making. Due to the subjectivity of individual assessments, experts can formulate different conclusions, sometimes the decisions received from experts contradict each other. In this case, it is necessary to coordinate the decision in any acceptable way. Perform discrepancy analysis, identify differences between expert assessments and establish their cause. One of the common methods of processing expert information is the method of paired comparisons by T. Saati [4; 5]. All alternatives and decision criteria are evaluated by constructing a matrix of pairwise comparisons. Each expert submits conclusions in the form of a square matrix, the elements of which are formed according to the scale of the relative weight of the alternative [7]. One of the tasks of T. Saati's method is the desire to reduce the influence of the human factor on the result. To determine the degree of correctness of the data in the completed matrix, the concept of the degree of consistency of the matrix was introduced.

Let's consider the procedure for coordinating expert evaluations for the task of synthesis of a fuzzy solution [10]. Let each k -expert perform the ranking by solving the multi-criteria problem of choosing alternatives $A = \{A_i\}, (i = 1 \div m)$ according to the criteria $K = \{K_j\}, (j = 1 \div n)$. For each k -expert, we have an individual matrix of pairwise comparisons of alternatives

$$A = (a_{mm}) = \begin{bmatrix} a_{11} & \dots & a_{1m} \\ \dots & \dots & \dots \\ a_{m1} & \dots & a_{mm} \end{bmatrix}. \quad (1)$$

If the judgments of the experts are agreed, then the matrix of paired comparisons has the following properties: it is inversely symmetrical (if $\alpha_{ik} = \beta$, then $\alpha_{ki} = 1/\beta$); diagonal elements are equal to one ($\alpha_{ii} = 1$); transitive ($\alpha_{is} \cdot \alpha_{sk} = \alpha_{ik}$, ($i = 1 \div m, k = 1 \div m, s = 1 \div m$)).

The consistency of the matrix means that the relative weights of the alternatives in the matrix of pairwise comparisons retain their value, regardless of how the comparison is performed. From a mathematical point of view, consistency is a linear dependence of the rows (columns) of the matrix (1).

Consistency of an inversely symmetric matrix of pairwise comparisons ($\alpha_{ik} = \beta$, $\alpha_{ki} = 1/\beta$), can be determined by the maximum eigenvalue of the matrix α_{max} with subsequent comparison of the obtained value with the order of the matrix ($\alpha_{max} \sim m$). The closer the value of the maximum eigenvalue of the matrix is to the order of the matrix, the more consistent the elements of the pairwise comparison matrix are. Complete consistency of the matrix A means equality $\alpha_{max} = m$. Then, the consistency condition of the matrix of pairwise comparisons can be written as follows:

$$A \cdot \bar{v} = \alpha \cdot \bar{v}, \quad (2)$$

where $\bar{v} = (v_1, v_2, \dots, v_i, \dots, v_m)$ – principal eigenvector of the matrix A ; α – the maximum eigenvalue of the matrix A .

Eigenvector \bar{v} matrices A , which ensures orderliness of relative evaluations of alternatives, is a measure of homogeneity of experts judgments.

In practical problems, the consistency of the matrix of pairwise comparisons may not be achieved, since the expert's judgment cannot be evaluated numerically. The inconsistency of the matrix of paired comparisons can be caused both by the personal preferences of the expert and by the degree of uncertainty of the evaluation object. The inconsistency of matrix A is the result of the interaction of these factors. Since uncertainty is a property of the object of assessment, the value of the homogeneity coefficient (CR) can be considered an indicator of the quality of the set of expert assessments.

$$CR = \frac{CI}{RI}, \quad (3)$$

where $CI = \frac{\alpha_{max} - m}{m - 1}$ – coefficient of homogeneity of the matrix A ;

$RI = \frac{1,98(m-2)}{m}$ – stochastic coefficient of homogeneity of the matrix A .

If $CR < 0,1$ – the level of homogeneity is acceptable, $CR > 0,1$ – it is necessary to agree the matrix of paired comparisons.

Given that pairwise comparison matrices are built on the basis of human judgment and assumptions, some degree of inconsistency between matrix elements is to be expected A . Such inconsistency is not critical, if it does not go beyond the defined limits [10].

The sequence of steps for checking expert evaluations for homogeneity can be formulated as follows.

1. Each expert evaluates alternatives according to criteria, as well as evaluates the priority of criteria based on pairwise comparisons, according to the selected scale.

2. The process of coordinating assessments includes discussion and argumentation of individual assessments, identification of common positions and the most well-founded judgments. For coordination, the Delphi method [2; 11] or its modifications can be applied.

3. The reconciliation process is terminated if the individual expert evaluations have converged to the acceptable interval of disagreement, or at least one of the experts does not agree to continue the process.

If the reconciliation process does not provide a tolerable level of discrepancy of individual assessments, then it may be appropriate:

- carry out normalization of individual evaluations by the average value;
- exclude extreme values from expert evaluations;
- consider the possibility of accepting non-agreed evaluations of alternatives, with subsequent analysis of the results of such a decision.

Conclusions. The essence of the method of expert evaluations consists in the logical and intuitive analysis of the internal and external environment of the organization as an object of management, the development of alternatives and the quantitative assessment of their quality. The generalized opinion of experts serves as a basis for making a choice. Complex use of intuitive and logical thinking, as well as formal processing of experts judgments allows to obtain indicators of the quality of alternatives when solving partially formalized problems of decision selection. The use of evaluations of alternatives obtained on the basis of experts conclusions is a means of taking into account the collective knowledge and experience of specialists in the conditions of unstructured, subjective information, the impossibility of applying deterministic algorithms for the justification of decisions.

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